GAS HEATING AND COOLING OF A FIXED TWO-COMPONENT BED OF SOLID PARTICLES

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Equations are obtained for the temperature variation of each material and of the gas for heating and cooling of a fixed bed composed of two materials with different thermophysical properties.

In the heating of materials in shaft furnaces, roasting of ferrous and nonferrous ores, and in similar technical processes, heat is applied to a bed of solid particles, consisting of a minimum of two materials possessing different thermophysical properties (mass heat capacity, density, etc.). The well-known solution for the Schumann problem [1] applies to a bed of uniform material.

It is evident that in a two-component bed of particles of the same size, the material with the smaller volume heat capacity will be heated faster by the gas than the material with the larger volume heat capacity. A temperature difference results between two neighboring particles of the different materials at the same horizontal level, causing a heat flux between the particles. Calculation shows that the temperature difference between particles at one horizontal level under certain conditions causes a heat flux tens of times larger than that due to the temperature difference along the channel formed between the lumps of material through which the gas flows. It has been shown by Chukhanov [2], that if the particles have no sharp projections or angularities, the heat flux component due to thermal conduction is negligibly small. The radiative component should not be neglected. In an exact calculation of radiative heat exchange between particles heated by a gas, a solution of the problem may be obtained, because of nonlinearity of the boundary conditions, only with the aid of electronic computers. Earlier work by the authors [3] has shown that, to an accuracy sufficient for engineering calculations, the law of thermal interaction between particles may be represented by Newton's law, in which the heat transfer coefficient α_{rad} is computed from the formula

$$\alpha_{\text{rad}} = 4\,\frac{C_{\Pi}}{100}\left(\frac{\overline{T}}{100}\right)^3. \qquad (1)$$

Accurate determination of C_{Π} for a bed of spherical particles is both laborious and complex. For simplicity it is expedient to assume that channels are formed in the bed in the direction of motion of the gas, the surface area of the channels being equal to the total surface area of the particles of both materials. The fraction of the channel surface area allotted to one of the materials will depend on its amount (percentage content in the mixture Π). Knowing the particle surface area, we may find the reduced radiation coefficient from the expression [4]

$$C_{\Pi} = C_0 \Big/ \left(\frac{1}{\varphi_{21}} + \frac{r_2}{a_2} + \frac{r_1}{a_1}\,\frac{f_2}{f_1}\right), \qquad (2)$$

where φ_{21} is the radiation coefficient of particles of the second material relative to the first. Its value is calculated on the assumption that the channel formed by the mixture of particles is a closed system of the two surfaces, for which

$$\varphi_{21} = \sin\frac{\pi\Pi}{100} \Big/ \pi\,\frac{100-\Pi}{100}. \qquad (3)$$

Taking into account the above-mentioned mutual heat exchange between particles, and considering a bed element of height dH and 1 m^2 in area, we may write the following equations to describe the heat transfer between the flowing gas and the fixed two-component bed:

for the first material

$$\varphi_1 c_1 \gamma_1 \frac{\partial t_1}{\partial \tau} = \alpha f_1 (t_g - t_1) + \alpha_{\text{rad}} (t_2 - t_1), \qquad (4)$$

for the second material

$$\varphi_2 c_2 \gamma_2 \frac{\partial t_2}{\partial \tau} = \alpha f_2 (t_g - t_2) - \alpha_{\text{rad}} (t_2 - t_1), \qquad (5)$$

for the gas stream

$$-wc\gamma \frac{\partial t_g}{\partial H} = \alpha f_1 (t_g - t_1) + \alpha f_2 (t_g - t_2). \qquad (6)$$

The boundary conditions of the problem are

$$H = 0, \quad t_g = T;$$
$$\tau = 0, \quad t_1 = t_2 = 0. \qquad (7)$$

Equations (4)–(6) hold for a bed whose materials have infinitely large thermal conductivity. The validity of this assumption is confirmed by the fact that in the majority of cases the values of Biot number calculated for a bed of solid particles prove to be less than 0.5.

Introducing the new variables

$$Y = \alpha f_1 H / wc\gamma, \qquad Z = \alpha f_2 \tau / \varphi_1 c_1 \gamma_1$$

and the notation

$$n = f_2/f_1, \qquad m = \varphi_1 c_1 \gamma_1 / \varphi_2 c_2 \gamma_2, \qquad A = \alpha_{\text{rad}} / \alpha f_1$$

we transform the system of equations (4)–(7) to the form

$$\frac{\partial t_1}{\partial Z} = (t_g - t_1) + A(t_2 - t_1), \quad (8)$$

$$\frac{\partial t_2}{mn\partial Z} = (t_g - t_2) - \frac{A}{n}(t_2 - t_1), \quad (9)$$

$$-\frac{\partial t_r}{\partial Y} = (t_g - t_1) + n(t_g - t_2) \quad (10)$$

with the following boundary conditions:

$$Y = 0, \quad t_g = T,$$

$$Z = 0, \quad t_1 = t_2 = 0. \quad (11)$$

We apply a Laplace-Carson transformation to (8) and (9). Then in the transforms we have

$$p\bar{t}_1 = \bar{t}_g - \bar{t}_1 + A(\bar{t}_2 - \bar{t}_1), \quad (12)$$

$$p\bar{t}_2 = mn(\bar{t}_g - \bar{t}_2) - mA(\bar{t}_2 - \bar{t}_1), \quad (13)$$

from which $\bar{t}_1$ and $\bar{t}_2$ may be found as functions of $\bar{t}_g$:

$$\bar{t}_1 = \frac{(p + mn + Am) + Amn}{(p + A + 1)(p + mn + Am) - A^2m}\bar{t}_g, \quad (14)$$

$$\bar{t}_2 = \frac{(p + A + 1)mn + Am}{(p + A + 1)(p + mn + Am) - A^2m}\bar{t}_g. \quad (15)$$

Equation (10) in transforms, taking account of (14) and (15), and after a number of transformations, takes the form

$$-\frac{\partial \bar{t}_g}{\partial Y} = \bar{t}_g(1 + n) -$$

$$-\bar{t}_g\frac{p(1 + mn^2) + mn(1 + n) + Am(1 + n)^2}{(p + A + 1)(p + mn + Am) - A^2m}. \quad (16)$$

The solution of this equation, satisfying the boundary conditions, is

$$\bar{t}_g = T\exp[-(1 + n)Y] \times$$

$$\times \exp\frac{p(1 + mn^2) + mn(1+n) + Am(1 + n^2)}{(p + A + 1)(p + mn + Am) - A^2m}Y. \quad (17)$$

To find the original of (17) it is necessary to transform the denominator of the exponent, reducing it to the following form:

$$(p - R_1)(p - R_2) = (p + A + 1)(p + mn + Am) - A^2m. \quad (18)$$

From this the roots of the quadratic equation may be found. They are

$$R_{1,2} = -\frac{1}{2}(mn + Am + A + 1) \pm$$

$$\pm \frac{1}{2}\sqrt{(mn + Am + A + 1)^2 - 4[mn + Am(1 + n)]}. \quad (19)$$

Finally, (17) may be transformed to the form

$$\bar{t}_g = T\exp[-(1 + n)Y]\exp\left[\frac{a}{p - R_1}Y + \frac{b}{p - R_2}Y\right], \quad (20)$$

where

$$a = \frac{(k + R_1)(1 + mn^2)}{R_1 - R_2}, \qquad b = \frac{(k + R_2)(1 + mn^2)}{R_2 - R_1},$$

$$k = \frac{mn(1 + n) + Am(1 + n)^2}{1 + mn^2}. \quad (21)$$

Using the formula (5)

$$F(p + \lambda) \to f(Z)\exp(-\lambda Z) + \int_0^Z \lambda f(\varepsilon)\exp(-\lambda\varepsilon)\,d\varepsilon \quad (22)$$

and the transition formula

$$\exp(\beta/p) \to I_0(2\sqrt{\beta Z}), \quad (23)$$

we obtain the original of the function for the gas stream temperature. In final form, the distribution of gas temperature in the bed is described by the equation

$$t_g = T\exp[-(1 + n)Y]\Big\{\exp(R_1Z)I_0(2\sqrt{aYZ}) -$$

$$-R_1\int_0^Z \exp(R_1\varepsilon)I_0(2\sqrt{aY\varepsilon})\,d\varepsilon + \int_0^Z\Big[\exp(R_1x)I_0(2\sqrt{aYx}) -$$

$$-\int_0^x R_1\exp(R_1\varepsilon)I_0(2\sqrt{aY\varepsilon})\,d\varepsilon\Big]\times$$

$$\times\sqrt{bY/(Z - x)}\exp[R_2(Z - x)]\,I_1(2\sqrt{bY(Z - x)})\,dx\Big\}. \quad (24)$$

To determine the temperature of the first material we use (8) and (9). Eliminating the unknown t_2, we obtain an equation for calculating t_1,

$$\frac{\partial t_1}{\partial Z} + t_1\left(1 + A\frac{1 + n}{n}\right) = \left(1 + A\frac{1 + n}{n}\right)t_g + \frac{A}{n}\frac{\partial t_g}{\partial Y}, \quad (25)$$

the solution of which has the form

Calculated Bed Temperatures

Method of calculation	Temperature	t, °C, for Z values of 0	1	3
According to equations (24), (26)–(28)	t_g	100	318	691
	t_1	0.0	150	532
	t_2	0.0	169	557
From the Schumann graphs for a uniform bed of equivalent heat capacity	t_g	100	315	688
	t_M	0.0	153	535

$$t_1 = \int_0^Z \left[\left(1 + A\frac{1+n}{n}\right)t_g + \frac{A}{n}\frac{\partial t_g}{\partial Y}\right] \times$$

$$\times \exp\left[\left(1 + A\frac{1+n}{n}\right)(\varepsilon - Z)\right] d\varepsilon. \quad (26)$$

Proceeding similarly with (9) and (10), for calculating t_2 we obtain the expression

$$t_2 = mn \int_0^Z \left[\left(1 + A\frac{1+n}{n}\right)t_g + \frac{A}{n}\frac{\partial t_g}{\partial Y}\right] \times$$

$$\times \exp\left[mn\left(1 + A\frac{1+n}{n}\right)(\varepsilon - Z)\right] d\varepsilon. \quad (27)$$

Determination of the integrals in (26) and (27) requires a knowledge of t_g and $\partial t_g/\partial Y$; t_g is determined from (24), and for $\partial t_g/\partial Y$ we obtain, respectively,

$$\frac{\partial t_g}{\partial Y} = -(1+n)t_g + T\exp\left[-(1+n)Y\right]\left[\sqrt{\frac{aZ}{Y}} \times\right.$$

$$\times \exp(R_1 Z) I_1(2\sqrt{aYZ}) -$$

$$- R_1\sqrt{\frac{a}{Y}}\int_0^Z \sqrt{\varepsilon}\exp(R_1\varepsilon) I_1(2\sqrt{aY\varepsilon})d\varepsilon +$$

$$+ \sqrt{ab}\exp(R_2 Z)\int_0^Z \sqrt{\frac{x}{Z-x}}\exp\left[(R_1 - R_2)x\right] \times$$

$$\times I_1(2\sqrt{aYx}) I_1(2\sqrt{bY(Z-x)})dx +$$

$$+ \exp(R_2 Z)\int_0^Z \exp\left[(R_1 + R_2)x\right] \times$$

$$\times I_0(2\sqrt{aYx})\Big\{b\left[I_0(2\sqrt{bY(Z-x)}) -\right.$$

$$\left.- I_1(2\sqrt{bY(Z-x)})/2\sqrt{bY(Z-x)}\right] +$$

$$+ I_1(2\sqrt{bY(Z-x)}) \cdot \frac{1}{2}\sqrt{\frac{b}{Y(Z-x)}}\Big\}dx -$$

$$- \sqrt{ab}\, R_1\exp(R_2 Z)\int_0^Z \sqrt{\frac{x}{Z-x}} \times$$

$$\times \exp(-R_2 x) I_1(2\sqrt{bY(Z-x)}) \times$$

$$\times \int_0^x \sqrt{\varepsilon}\exp(R_1\varepsilon) I_1(2\sqrt{aY\varepsilon})d\varepsilon dx - R_1\exp(R_2 Z) \times$$

$$\times \int_0^Z \Big\{b\left[I_0(2\sqrt{bY(Z-x)}) -\right.$$

$$\left.- I_1(2\sqrt{bY(Z-x)})/2\sqrt{bY(Z-x)}\right] +$$

$$+ \frac{1}{2}\sqrt{\frac{b}{Y(Z-x)}} I_1(2\sqrt{bY(Z-x)})\Big\}\exp(R_2 x) \times$$

$$\times \int_0^x \exp(R_1\varepsilon) I_0(2\sqrt{aY\varepsilon})d\varepsilon dx. \quad (28)$$

For numerical calculation in not particularly important cases, the curve $\partial t_g/\partial Y = f(Z)$ may be constructed from the graphs of Schumann et al. [5].

When one of the materials is not present in the bed and there is no heat transfer between particles, i.e., when $n = 0$, $A = 0$, equations (24) and (26)–(28) transform to the well-known equations describing heating of a single material [6].

Analysis of the original differential equations allows a more sharply drawn conclusion regarding preferential heating of one of the materials. In fact, for time zero, as follows from (8) and (9), the relation

$$\frac{\partial t_2}{\partial Z} : \frac{\partial t_1}{\partial Z} = mn = \frac{f_2}{\varphi_2 c_2 \gamma_2} : \frac{f_1}{\varphi_1 c_1 \gamma_1}$$

is valid, from which it is seen that the rate of heating, for example, of the second material, will in general be the greater, the greater the surface area corresponding to unit volume heat capacity of the material.

On the basis of the formulas obtained, we calculated the heating of a bed consisting of ore (90% by weight) and coke (10%) by a gas at a temperature of 1000° C. The bed is composed of near-spherical particles and with a mean diameter of 0.005m. The heat capacities of 1 m^3 of ore and coke are, respectively, 1280 and 910 $kJ/m^3 \cdot$degree, and that of the gas is 1.17 $kJ/m^3 \cdot$degree. The gas velocity, referred to the free shaft section, is 0.5 m/sec. The bed initial temperature is 0° C. The calculated coefficient of convective heat transfer turned out to be 87.3 $W/m^2 \cdot$degree, while the coefficient of heat transfer between particles was 84.8 $W/m^2 \cdot$degree. The results of the calculation for $Y = 2.0$ are shown in the table as a function of Z. Also given are the results of calculations according to the Schumann graphs for a uniform bed possessing a volume heat capacity equivalent to the bed of two materials. For a uniform bed $Y_0 = 2.53$, and the relation between Z and Z_0 has the form $Z_0 = 1.063\,Z$.

Comparison of the results obtained shows that for approximate calculations it is possible to recommend the following scheme for determining the temperatures t_g, t_1 and t_2 in a two-component bed with the amount of one of the components $\le$10%: first, the parameters Y_0 and Z_0 are calculated for a uniform bed of equivalent heat capacity. Then from the graphs of [6] the quantities t_g and $\partial t_g/\partial Y$ are determined, and the temperatures t_1 and t_2 are computed from equations (26) and (27).

When the content of the second material in the bed is large, and in accurate calculations, it is necessary to use formulas (24), (26), and (27).

NOTATION

φ_i is the fraction of material in 1 m^3 of bed; f_i is the surface area of material in 1 m^3 of bed; Π is the percentage content of material in mixture; C_Π, C_0 are the radiation coefficients; a_i is the absorptivity of particle material; r_i is the reflectivity of particle material; c_i, c is the mass heat capacity of material and gas; γ_i, γ are the bulk density of material and gas density; ω is the gas velocity; t_i, t are the temperature of material and gas; $\overline{T}$ is the mean gas temperature; α is the coefficient of heat transfer from gas to surface of materials; α_{rad} is the coefficient of mutual heat transfer between particles; H is the bed height; τ is the time; $Y = \alpha f_1 H/wc\gamma$ is the bed height parameter; $Z = \alpha f_1 \tau/\varphi_1 c_1 \gamma_1$ is the time parameter. Subscripts 1, 2 indicate that the parameters belong to the first or to the second material.

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